Influence of high-frequency laser frequency noise on the stability of an optical clock

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***Abstract*—Neutral atom optical frequency standards have the potential to reach extremely high stabilities due to a high number of atoms. The frequency noise of the interrogation laser can lead to a significant degradation of the stability of a frequency standard through aliasing (Dick effect). Thus, to exploit the potential of the frequency standards, the spectral density of the frequency noise of the laser must be sufficiently low. In this paper we discuss the influence of the high-frequency noise in an optical clock based on ultracould calcium atoms and show how the high-frequency laser frequency noise can be efficiently suppressed by using the reference optical cavity as a filter.**

I. Introduction

Optical frequency standards with laser-cooled neutral atoms are now reaching stabilities higher than the microwave clocks due to the high numbers of atoms (10^6 – 10^7) and narrow clock transitions involved. To realize an optical clock with ballistic calcium atoms [1], a four-pulse Ramsey-Bordé atom interferometer in time domain is used [2], which consists of two counter-propagating pairs of pulses acting as beam splitters. This method achieves high spectral resolution determined by the time T between the two pulses of a pair. At the same time, the whole velocity distribution of the atomic ensemble contributes to the signal as the large Fourier width of a single pulse of the duration T_p covers the full Doppler width. Since the pulses are a few microseconds long, the high-frequency laser frequency noise with the Fourier frequencies up to the MHz-range contributes to the detection noise and degrades the stability of the atomic clock through aliasing (Dick effect) [3].

Due to the Dick effect the present stability of the optical clock with ultracold calcium atoms is two orders of magnitude above the ultimate stability given by the quantum projection noise [4]. Thus, to improve the stability it is essentially to reduce the laser frequency noise. In the low-frequency range the frequency noise is mostly due to acoustic vibrations, which we could sufficiently suppress by a vibration-insensitive cavity design [5]. The reduction of the high-frequency noise in the range up to several MHz, which is high for diode lasers compared to other laser types, is limited by the gain and the bandwidth of the feed-back loop. Using an optical cavity in transmission as a low-pass filter can efficiently suppress laser noise at higher frequencies [6], [7].

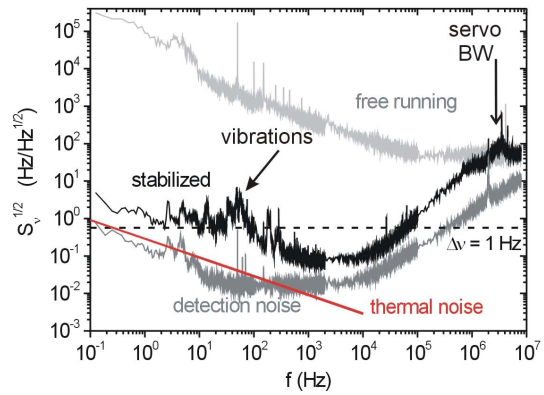

Figure 1. The spectrum of frequency fluctuations of the interrogation laser.

II. Interrogation Laser

The laser to interrogate the clock transition $^1S_0$ – $^3P_1$ in ^{40}Ca at 657 nm is an extended-cavity diode laser that is stabilized to a highly stable reference cavity described in detail elsewhere [8]. The linewidth of the laser of about one Hertz was determined by recording the beat note between two identical but independent systems [8]. Fig. 1 shows the spectrum of frequency fluctuations of the interrogation laser. The fluctuations of the free running laser (grey line) are significantly suppressed using an active stabilization to a reference cavity (black line). The servo electronic is in principle able to reduce the frequency noise to the shot noise level of 0.1 Hz/Hz$^{1/2}$ in the frequency range between one and ten kilohertz. The increase of the frequency fluctuations above ten kilohertz is due to reduced gain of the servo loop at higher frequencies because of a unity-gain frequency of about 3 MHz. The increase of frequency fluctuations at frequencies

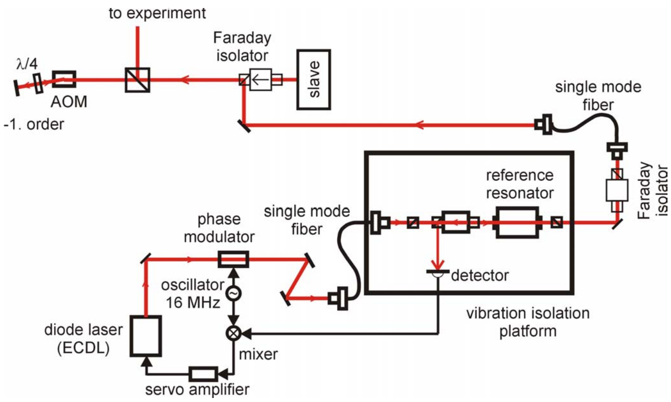

Figure 2. Schematic setup for filtering laser light with an optical cavity.

below one kilohertz is caused by fluctuations of the optical length of the reference cavity due to acoustic and seismic vibrations acting on the cavity. We have now implemented a vibration insensitive reference cavity, which allows to reduce the frequency noise in the low frequency range [5].

III. Suppression of High-Frequency Noise

A. Noise Suppression by Optical Filtering

Optical cavities are widely used for spectral and spatial cleaning of laser beams [6], [7]. The light transmitted through a cavity can not follow fast frequency fluctuations due to the storage time of the cavity. Thus, the cavity acts as a low pass filter and suppresses the spectral power density of frequency fluctuations S_ν^0 of the incident light to S_ν^c as follows

$$S_\nu^c(f) = S_\nu^0(f)\cdot\frac{1}{1+(f/\Delta\nu_{\text{cav}})^2}, \tag{1}$$

where $\Delta\nu_{cav}$ is the cavity resonance half width at half maximum.

B. Experimental Setup

The experimental setup for filtering laser light with an optical cavity is shown in Fig. 2. As a filtering cavity the reference cavity itself is used with $\Delta\nu_{\text{cav}} = 9.5\,\text{kHz}$. From the total output power of the laser of 6 mW, 0.45 mW are used for the Pound-Drever-Hall stabilization. On resonance the cavity transmits about 9% of the incident power resulting in 40 µW transmitted power. Since this power is not sufficient for the experiment, an additional slave laser is injection locked to provide a power of 30 mW.

C. Noise of the Injection-Lock

The light field of an injection-locked laser follows the frequency noise fluctuations of the master laser and exhibits the linewidth of the master field [9]. Furthermore the slave laser adds an additional noise due to its own spontaneous emission. The spectral power density of frequency fluctuations of the slave field S_ν^{SL} can be written for $f < \Delta f_{lock}$ according to [9] as

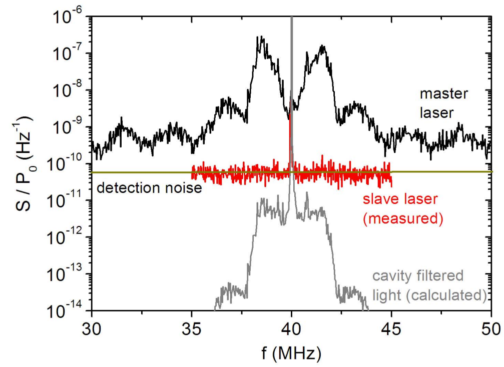

Figure 3. Spectra of the fluctuations of the photodiode current taken with original light (master laser, upper line) and with slave laser injection-locked with cavity filtered light (middle line). The lower line depicts the calculated values according to (1) neglecting the additional noise due to the injection-lock.

$$S_\nu^{SL}(f) = S_\nu^{ML}(f) + S_\nu^{free}(f)\frac{f^2}{\Delta f_{lock}^2}, \tag{2}$$

where $S_\nu^{ML}(f)$ and $S_\nu^{free}(f)$ are the spectral power density of frequency fluctuations of the master field and of the free running slave field respectively, Δf_{lock} is the half-locking bandwidth. For further calculations, the typical experimental values of $S_\nu^{free} = 3\cdot 10^6\,\text{Hz}^2/\text{Hz}$, corresponding to a free running linewidth of the slave of 10 MHz, and $\Delta f_{lock} = 200\,\text{MHz}$ are taken.

D. Self-Heterodyne Measurements

To characterize the wide-band frequency noise of the laser before and after the modifications, the delayed self-heterodyne technique was used [10]. The idea of this technique is to convert frequency fluctuations of the laser into variations of light intensity in a Mach-Zehnder interferometer. The laser beam is split on a beam splitter, one part is frequency shifted by an acousto-optical modulator, sent through 80 m of fibre and superimposed with the un-shifted laser beam on a photodiode. The power spectrum of the photodiode signal is recorded with a spectrum analyzer. The spectral density S normalized by the carrier power P_0 is proportional to the spectral power density of frequency fluctuations S_ν.

Fig. 3 shows the enormous improvement of the spectral purity of the cavity filtered light. The measured spectrum of the fluctuations of the slave laser injection-locked with the cavity filtered light is now limited by the noise and the dynamic range of the spectrum analyzer. The expected improvement according to (1) is presented in the lowest line without taking into account the additional noise due to injection-lock.

IV. Laser Performance

The frequency stability and the linewidth of the laser system is determined in a beat measurement with a second laser. The lasers, which are compared, should have similar properties since the beat signal is dominated by the laser with

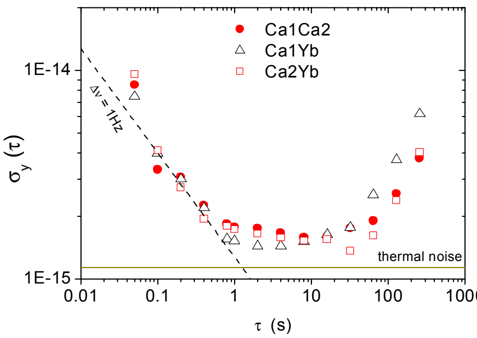

Figure 4. Allan standard deviations of the measured beat frequencies. The solid line indicates the calculated thermal noise level.

poorer properties. The two calcium lasers at the frequency of 456 THz (named Ca1 and Ca2 lasers) can be compared directly by measuring the beat frequency, which is in rf range.

At PTB, an other highly stable laser (in the following named Yb laser) is available, which is used as a clock laser in an $^{171}Yb^{+}$-ion frequency standard. Since the frequency of the Yb laser is 344 THz, a femtosecond frequency comb is used for the comparison with the calcium lasers. To measure the beat between the Ca1 and the Yb laser, the transfer-oscillator principle is used [11]. From the simultaneously measured beat frequencies between the Ca1 and Yb lasers and the Ca1 and Ca2 lasers the beat frequency between the Ca2 and Yb lasers can be calculated.

Fig. 4 shows the Allan standard deviations, which are calculated from the measured beat frequencies. All lasers were locked to their reference cavities. The linear drift of the respective cavities is subtracted. All three Allan deviations exhibit very similar behavior. The $1/\tau^{1/2}$ decrease for $\tau < 1$ s corresponds to the white frequency noise level of 1 Hz laser linewidth. The flicker frequency floor for $1\text{ s} < \tau < 30$ s lies slightly above the calculated thermal noise level [12], which gives the fundamental limitation of the frequency stability.

V. Dick Effect

The Dick effect describes the aliasing of the frequency noise S_y of the interrogation laser at harmonics of the cycle frequency f_c due to the non-continuous interrogation of the atoms [3]. The Dick effect for an optical frequency standard using Ramsey-Bordé interrogation was first addressed in [13]. The contribution of the aliasing effect to the stability of a frequency standard is given by

$$\sigma_y^2(\tau) = \frac{1}{\tau}\sum_{k=1}^{\infty}\left|\frac{g_k}{g_0}\right|^2 S_y\left(\frac{k}{T_c}\right), \qquad (3)$$

where g_k and g_0 are the zeroth and k^{th} Fourier component of the sensitivity function $g(t)$ describing the sampling of the laser frequency by the atoms, and T_c is the cycle time.

The contribution of the Dick effect to the stability of the optical clock is calculated according to (3) with the known spectral properties of the interrogation laser and the filtered light for different pulse separation times T. The pulse width amounts to $T_p = 1$ µs the cycle time $T_c = 50$ ms.

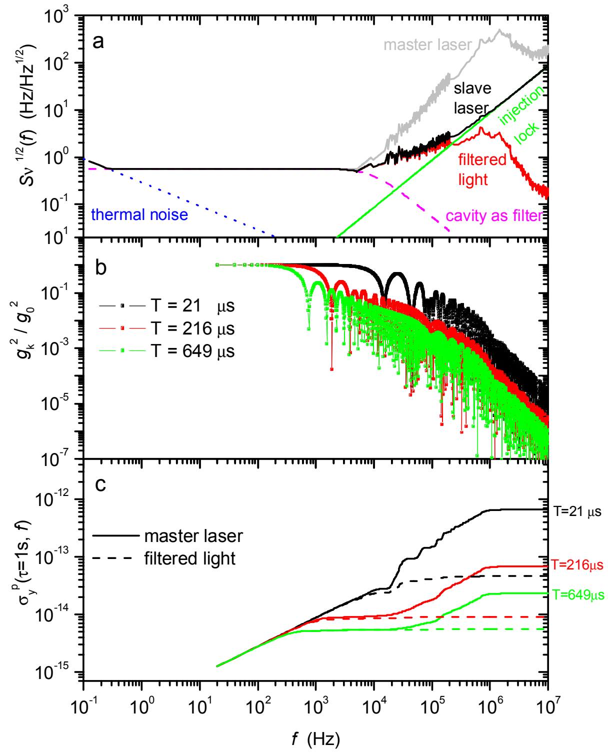

Figure 5. a) spectral density of frequency fluctuations of the master laser (grey) and of the slave laser (black), which is injection-locked with the cavity filtered light of the master. b) calculated coefficients of the sensitivity function g_k/g_0 for different pulse separation times T in 4-pulse Ramsey-Bordé atom interferometer. c: partial sum of contributions to the Dick effect for different lasers and separation times (see text).

Fig. 5a shows the measured spectral density of frequency fluctuations of the master laser (grey line). The calculated improvement due to the optical filtering according to (1) is depicted in red. The frequency fluctuations of the slave laser (black line), which is injection-locked with the cavity filtered light of the master laser, is calculated with taking into account the additional noise caused by the injection-locking itself according to (2) (green line). For all curves, white S_ν in the low frequency range was assumed as calculated from the laser linewidth $\Delta\nu$ determined in the beat measurement using $\Delta\nu = \pi S_\nu$.

Fig. 5b shows the calculated coefficients of the sensitivity function g_k normalized by g_0 for different pulse separation times T of the four-pulse Ramsey-Bordé atom interferometer. g_k/g_0 shows three different slopes: Starting with the value of one, it changes the slope to -1 at the frequency $f = 1/(2T)$. At the frequency $f = 1/(2T_p)$ it rolls off as f^{-2}.

The value $\sigma_y^p(\tau, f_{\max})$ shown in Fig. 5c is the partial sum of (3) taken up to a maximum index $k_{\max} = f_{\max}T_c$, i.e. considering only frequency noise $S_y(f)$ up to a maximum frequency $f_{\max}$. This calculations show that the reduction of the frequency fluctuations in the high-frequency range can enormously improve the stability of an optical clock (e.g. for $T = 216$ µs, $T_p = 1$ µs, and $T_c = 50$ ms from $\sigma_y(1\text{s}) = 7\cdot10^{-14}$ in case of the master laser to $\sigma_y(1\text{s}) = 9.1\cdot10^{-15}$ for the filtered light). It is also advantageous to use long separation times T,

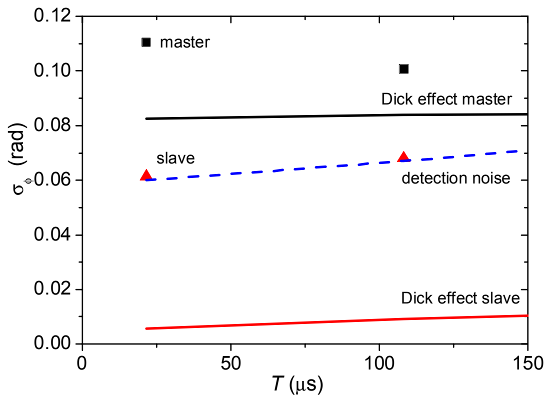

Figure 6. Noise of the measured excitation probability expressed as a standard deviation of the interferometric phase on the slope of Ramsey-Bordé fringes for different pulse separation times T with master laser (squares) and with slave laser (triangles). The dashed line indicates the detection noise averaged from measurements on top and bottom of the fringes. The solid lines are the calculation of the Dick effect according to (3).

since in this case the sensitivity function starts to drop at lower frequencies, which, however, is limited by the 430 µs lifetime of the excited state.

VI. Noise of Ramsey-Bordé Interrogation

To demonstrate the improvement due to better spectral properties of the interrogation laser, we performed atom interferometric measurements on an ensemble of $4{\cdot}10^7$ ultracold calcium atoms with a temperature of 20 µK. The rms-noise of the mean excitation probability p_e was measured on the top, bottom, and maximum slope of the central fringe for different pulse separation times T. The noise at the top and bottom of the Ramsey fringes is to first order insensitive to frequency fluctuations of the laser and contains all noise sources except the Dick effect. If the frequency noise of the interrogation laser is white, then the additional noise on the slope of the fringe is only due to the Dick effect. The measured rms-noise of p_e is converted to the noise of the interferometric phase σ_φ and shown in Fig. 6. The dashed line gives the average noise measured on the top and bottom of the fringe. The solid lines indicate the calculated contributions of the Dick effect. The measured noise is well represented by the quadratic sum of the Dick effect and the non-frequency dependent noise. However, the reduction of the Dick effect by a factor of 10 could not be observed in the measured noise since the measurements with the slave laser are now limited by the detection noise [14].

VII. Conclusion

We investigated the influence of the high-frequency laser frequency noise on the stability of an optical clock. Filtering the light of the interrogation laser using an optical cavity efficiently reduces the frequency noise in this range and decreases the influence of the Dick effect by one order of magnitude. At this level, now the noise of the interferometric signal is limited by the frequency independent noise of the detection.

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